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Question Paper Code : 10877

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A and B are mutually exclusive (assuming neither A nor B has zero probability) will they be independent? Justify your answer.
2. Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of three men selects his own hat?
3. Starting at 5:00 AM, every half hour there is a flight from San Francisco airport to Los Angeles International airport. Suppose that none of these planes is completely sold out and they always have room for passengers. A person who wants to fly to Los Angeles. arrives at the airport at a random time between 8:45 AM and 9:45 AM. Find the probability that she waits at most 10 minutes.
4. Let X is a random variable. Is the statement $X + X = 2X$ true? Justify your answer.
5. Define Stochastic Processes. Draw sample paths of discrete state space and discrete index set
6. A computer system can operate in two different modes. Every hour it remains in the same mode or switches to a different mode according to the transition probability matrix $P = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$

If the system is in mode I at 5.30 pm, what is the probability that it will be in mode I at 8.30 pm on the same day?

7. Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day. What is the expected time until the tenth immigrant arrives?
8. Show that W is smaller in an M/M/1 model having arrivals at rate λ and service at rate 2μ than it is in a two-server M/M/2 model with arrivals at rate λ and with each server at rate μ .
9. Is birth and death process a continuous time Markov chain? Justify.
10. Why do we need queueing theory? What are the performance measures of a queueing model?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Suppose that an airplane engine will fail, when in flight, with probability $(1-p)$ independently from engine to engine; suppose that the airplane make a successful flight if at least 50% of its engines remain operative. For what values of p is a four-engine plane preferable to two-engine plane? (6)
- (ii) Two movie theatres compete for 900 visitors. Suppose each visitor chooses one of the two halls independent of the choice of the other visitors; how many seats should each theatre have so that that probability of turning away any visitor for lack of seats is less than 1%. (10)

Or

- (b) (i) Let X be a nonnegative continuous random variable with distribution function F . Prove that $E[X] = \int_0^{\infty} [1 - F(t)] dt$. (6)
 - (ii) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom. Assuming that the prisoner will always select doors 1, 2 and 3 with probabilities 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom? (10)
12. (a) (i) Suppose the joint probability density function of two continuous random variables X and Y is constant over the region $x^2 + y^2 \leq 4$. Determine $P[X^2 + Y^2 \leq 1]$. (8)
 - (ii) Suppose that X and Y are two independent exponential random variables with parameters λ_1 and λ_2 respectively. Compute
 - (1) $\Pr[X < Y]$
 - (2) the distribution functions of $\min(X, Y)$ and $\max(X, Y)$. (8)

Or

- (b) (i) Two methods of measuring surface smoothness are used to evaluate a paper product. The measurements are recorded as deviations from the nominal surface smoothness in coded units. The joint probability distribution of the two measurements is a uniform distribution over the region $0 < x < 4$; $0 < y$ and $x-1 < y < x+1$. Determine the following: (1) $P(X < 0.5, Y < 0.5)$ (2) $E[X]$. (4 + 4)
- (ii) In the transmission of digital information, the probability that a bit has high, moderate and low distortion is 0.01, 0.10 and 0.95 respectively. Suppose that three bits are transmitted and that amount of distortion of each bit is assumed to be independent. Let X and Y denote the number of bits with high and moderated distortion out of the three, respectively. Determine (1) $E[X]$ (2) $E[Y/X = 1]$ (3) Are X and Y independent? (3 + 3 + 2)
13. (a) (i) Three white and three black balls are distributed in two urns in such a way that each contains three balls. We say that the system is in state i , $i = 0, 1, 2, 3$, if the first urn contains i white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let X_n denote the state of the system after the n th step. Explain why $\{X_n, n = 0, 1, 2, \dots\}$ is a Markov chain and calculate its transition probability matrix. (6)
- (ii) Consider a Markov chain $(X_n)_{n \geq 0}$ on the state space $\{0, 1, 2, 3\}$ with transition probability matrix P given by
- $$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0.3 & 0 & 0.7 & 0 \\ 0.4 & 0.6 & 0 & 0 \end{bmatrix}$$
- (1) Draw the graph of this chain. Is the chain reducible? (2)
 (2) Find the recurrent, transient, and absorbing state (s) of this chain. (3)
 (3) Compute the fraction of time spent at state 0 in the long run. (2)
 (4) On the average, how long does it take to reach state 0 after starting from state 2? (3)
- Or
- (b) (i) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7 ; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Model this problem as a discrete time Markov chain. (6)

- (ii) A Markov chain has states $\{0, 1, 2, 3\}$ and transition probabilities

$$P_{ij} = \begin{cases} 1 & i=0; j=1 \\ \frac{1}{1+i} & i>0; j=0 \\ \frac{i}{i+1} & i>0; j=i+1 \end{cases}$$

Sketch the Markov Chain. Find the communicating classes. (10)

Which state are transient, null recurrent and positive recurrent?

14. (a) (i) Consider a machine that works for an exponential amount of time having mean $\frac{1}{\lambda}$ before breaking down; suppose that it takes an

exponential amount of time having mean $\frac{1}{\mu}$ to repair the machine.

If the machine is in working condition at time 0. then what is the probability that it will be working at time $t=10$? (6)

- (ii) Derive the steady state probabilities of a birth and death process. Hence obtain the steady state probability of being n number of customers in the system of an M/M/1 queue. (10)

Or

- (b) (i) Machine 1 is currently working. Machine 2 will be put in use at a time t from now. If the lifetime of machine i is exponential with rate $\lambda_i, i=1,2$ what is the probability that machine 1 is the first machine to fail? (6)

- (ii) Explain M/M/1 Queueing model. Derive the waiting time distribution. Establish Little's formula. (10)

15. (a) (i) Differentiate between open and closed queueing networks (6)

- (ii) Consider two machines, both of which have an exponential life time with mean $\frac{1}{\lambda}$. There is a single repairman that can service machines at an exponential rate μ . Set up the Kolmogorov backward equations and solve. (10)

Or

- (b) (i) Define and explain tandem queueing model in detail. (6)

- (ii) Derive Pollaczek-Khinchine formula for a M/G/1 queueing model. (10)